

## Technical Note

# A Note on the Problem of Shear Localization during Chip Formation in Orthogonal Machining

J. Huang and E.C. Aifantis

## Keywords

adiabatic shear banding, chip formation, Ti-6Al-4V

THE PURPOSE of this note is to elaborate on an important aspect of shear localization during chip formation. This study is motivated by a paper recently published in this journal (Ref 1). The aforementioned aspect is concerned with the prediction and the role of shear band spacing during discontinuous or serrated chip formation. The results in Ref 1 are based on a simplified maximum shear stress criterion, which leads to a mechanical model predicting the onset of shear localization in the chip. No information is provided, however, on shear band widths and spacings, even though some experimental data on shear band spacings are provided in Ref 2. Additional information and theoretical results on such features can be obtained by exploring a gradient theory for the localization of deformation (Ref 3-5).

For the problem of adiabatic shear banding, which is closely related to the problem of localized chip formation, a detailed analysis of the critical condition and shear band width is provided in Ref 6 on the basis of the aforementioned gradient theory. These results are cast here in a form appropriate to discuss the conditions and associated features pertaining to serrated chip formation, as follows.

Critical localization condition:

$$H_c = -\left(c + \frac{ks}{\rho c_v}\right)q^2$$

Internal length scale ( $l \sim 1/q$ ):

$$l = \sqrt{-\frac{1}{H_c} \left(c + \frac{ks}{\rho c_v}\right)}$$

Shear band width:

$$w \propto l$$

Thus, for a meaningful nonzero wave number  $q$ , the instability occurs at the softening region of the stress-strain curve ( $H_c < 0$ ). These results are obtained from the characteristic equa-

tion listed in Ref 6 for the linear stability analysis of localized shear deformation

$$w^3 + A\omega^2 + B\omega + C = 0$$

where

$$A = [q^2(k + c_v s) - \Phi \dot{\gamma}] / (\rho c_v)$$

## Nomenclature

$\tau$	Shear stress
$g$	Shear strain
$\dot{\gamma}$	Shear strain rate
$q$	Temperature
$c$	Force-like strain gradient coefficient
$r$	Mass density
$k$	Thermal conductivity
$c_v$	Specific heat
$H = \frac{d\tau}{d\gamma}$	Slope of the stress-strain curve
$h = \frac{\partial \tau}{\partial \gamma}$	Strain hardening coefficient
$s = \frac{\partial \tau}{\partial \dot{\gamma}}$	Strain rate sensitivity
$\Phi = \frac{\partial \tau}{\partial \theta}$	Thermal softening coefficient
$\mu = \frac{1}{\tau} \frac{\partial \tau}{\partial \gamma}$	Normalized strain hardening coefficient
$m = \frac{\partial \ln \tau}{\partial \ln \dot{\gamma}}$	Strain rate hardening coefficient
$f$	Feed rate
$j$	Shear zone angle
$a$	Rake angle
$V_c$	Chip flow velocity
$V$	Cutting velocity
$D_y$	Thickness of shear zone

J. Huang and E.C. Aifantis, Center for Mechanics of Materials and Instabilities, Michigan Technological University, Houghton, MI 49931, USA.

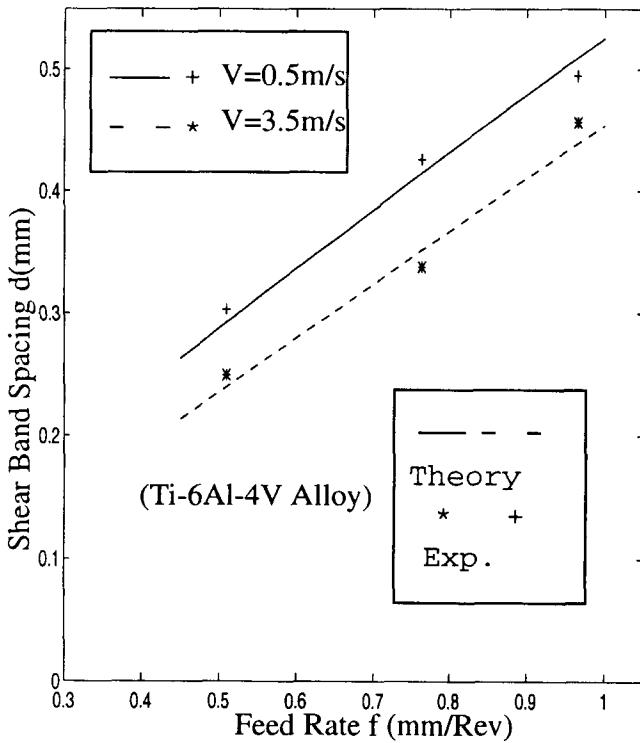


Fig. 1 Effect of feed rate on shear band spacing at different cutting velocities

$$B = q^2 [q^2 ks + \rho c_v (H + cq^2)] / (\rho^2 c_v)$$

$$C = q^4 k (h + cq^2) / (\rho^2 c_v)$$

By utilizing the above characteristic equation, one can determine the preferred wave number  $q_p$  for which the corresponding growth rate  $\omega_p > 0$  is a maximum. In fact, the following inequality can be established

$$0 \leq \omega_p \leq -\frac{c_v H}{k + c_v s}$$

The characteristic time for the formation of shear localization  $t_c$  is then estimated as

$$t_c \sim \frac{1}{\omega_p} \geq \frac{k + c_v s}{c_v H} \sim \frac{s}{H} = \frac{m\tau}{H\gamma}$$

According to the cutting configuration (Ref 7), we have

$$\gamma = \frac{\cos \alpha}{\cos(\phi - \alpha)} \frac{V}{\Delta y}, \quad V_c = \frac{\sin \phi}{\cos(\phi - \alpha)} V$$

where  $\Delta y \propto f$ .

It follows that the shear band spacing  $d \sim V_c t_c$  can be estimated by the relation

$$d = \chi_1 \frac{mf \sin \phi}{\lambda \cos \alpha}$$

The quantity  $\lambda$  is a normalized parameter, similar to the so-called flow localization parameter. It is defined as

$$\lambda = -\frac{H}{\tau} = -\left[ \mu + \left( \frac{1}{\tau} \frac{d\theta}{d\gamma} \right) \Phi \right]$$

By following the same procedure as in Ref 1 for the flow localization parameter, it can be shown that  $\lambda$  can be expressed in terms of the material properties and cutting conditions as

$$\lambda = -\left[ \frac{n}{\gamma} + \frac{\beta \Phi}{\rho c_v (n+1) \left( 1 + 1.328 \sqrt{\frac{k\gamma}{Vf}} \right)} \right. \\ \left. n+1 - \frac{0.664 \sqrt{\frac{k\gamma}{Vf}}}{1 + 1.328 \sqrt{\frac{k\gamma}{Vf}}} \right]$$

Figure 1 shows the effects of feed rate on the shear band spacing for Ti-6Al-4V at different cutting velocities. The microstructure of the uncut Ti-6Al-4V alloy consists of a lamellar  $\alpha$  structure with an intergranular  $\beta$  structure (Ref 2). It is found that the predicted shear band spacing is in agreement with the experimental data (Ref 2).

## References

1. J.Q. Xie, A.E. Bayoumi, and H.M. Zbib, Analytical and Experimental Study of Shear Localization in Chip Formation in Orthogonal Machining, *J. Mater. Eng. Perform.*, Vol 4 (No. 1), 1995, p 32-39
2. A.E. Bayoumi and J.Q. Xie, Some Metallurgical Aspects of Chip Formation in Cutting, *Mater. Sci. Eng.*, Vol A190, 1995, p 173-180
3. E.C. Aifantis, On the Microstructural Origin of Certain Inelastic Models, *J. Eng. Mater. Technol. (Trans. ASME)*, Vol 106, 1984, p 326-330
4. E.C. Aifantis, The Physics of Plastic Deformation, *Int. J. Plast.*, Vol 3, 1987, p 211-247
5. E.C. Aifantis, Spatio-temporal Instabilities in Deformation and Fracture, *Computational Material Modeling*, ADV.42/PVP-V.294, ASME, 1994, p 199-222
6. H.T. Zhu, H.M. Zbib, and E.C. Aifantis, On the Role of Strain Gradients in Adiabatic Shear Banding, *Acta Mech.*, Vol 111, 1995, p 111-124
7. P.L.B. Oxley, *The Mechanics of Machining: An Analytical Approach to Assessing Machinability*, Ellis Horwood, Chichester, West Sussex, U.K., 1989